The intensity fluctuation distribution of laser light

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even and odd parts with respect to the time τ, we find:

$$-\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial\tau^2}-\nu^2 D_5{}^2\right)Q = 2Q\frac{\partial}{\partial r}D_5 Q+2H\frac{\partial}{\partial r}D_5 H+\frac{\partial}{\partial r}\frac{\partial G_1}{\partial\tau}-\nu\frac{\partial}{\partial r}D_5 G_2+\frac{\partial}{\partial r}\times\left(5+r\frac{\partial}{\partial r}\right)I_2' \tag{6}$$

$$-\left(\frac{\partial^2}{\partial\tau^2}-\lambda^2 D_5{}^2\right)H = 2QD_5 H+2HD_5 Q+2\frac{\partial Q}{\partial r}\frac{\partial H}{\partial r} \tag{7}$$

where $G_1+G_2 = G$; G_1, G_2 are the odd and even parts respectively, and I_2' is the even part of I'.

One immediately remarks that if we leave out the external force terms, the equations (6), (7) reduce to Chandrasekhar's equations. We conclude this letter by pointing out that the equations deduced by us generalize the well-known equations for the second-order correlation functions for the velocity and the magnetic field.

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The intensity fluctuation distribution of laser light

Abstract. An experimental investigation of laser noise by measurement of photon-counting distributions over a range of sample times and pumping levels is reported. These results are compared with theoretical predictions and show good agreement. In addition a simple approximate expression for the second moment of the intensity fluctuations is given.

In a recent paper Lax and Zwanziger (1970) computed the intensity fluctuation distribution of integrated laser light near threshold. They remarked that detailed measurements of $p(m, T)$, the photon counting distribution for arbitrary sample time T, had not yet been reported though several workers have made measurements with short sample times (e.g. Freed and Haus 1966, Armstrong and Smith 1965, Arecchi *et al.* 1967, Chang *et al.* 1967, Pike 1969). In this letter we present a set of such detailed noise measurements for a Spectra Physics model 119 single-mode gas laser. Contact is made with the above computations through the factorial moments of $p(m, T)$, which are the actual moments of the intensity fluctuation distribution. A simple approximate analytic expression for the second factorial moment is given which gives results indistinguishable from the full theory. We compare the experimental values for these moments with detailed results of Lax and Zwanziger (1970).

The experimental techniques for measuring $p(m, T)$ are now standard (Johnson *et al.* 1966 a). A low dark-current FW130 photomultiplier was used with a resolving time of 40 ns and an uncooled dark count rate of 25 counts per second. A solid angle corresponding to a small fraction of a coherence area (6%) was subtended by the detector giving a count rate at threshold of $1{\cdot}5 \times 10^5$ counts per second corresponding to a photon flux from the laser of 3×10^8 per second. The laser cavity was detuned until it could be taken through threshold by varying its length by means of a piezo-electric mirror mount. A slow servo loop maintained the output amplitude constant by controlling the same mirror. Between 10^8 and 10^5 samples were taken for each

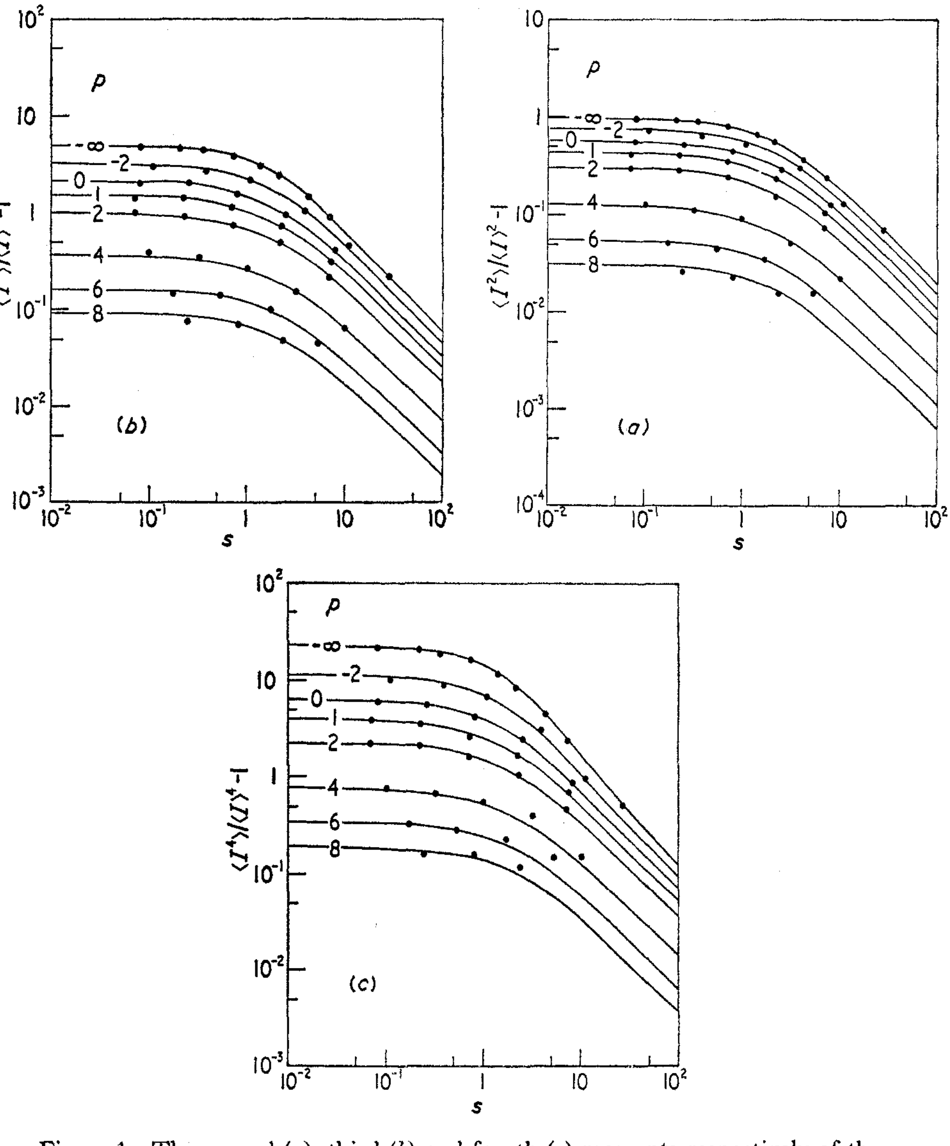

Figure 1. The second (*a*), third (*b*) and fourth (*c*) moments respectively of the intensity fluctuations of a laser near threshold obtained from measurements of the photon-counting distribution $p(m, T)$ for various sample times T. The continuous curves are from the theory of Lax and Zwanziger.

distribution, depending on the sample time. The greatest limitation on the accuracy of the results was imperfect acoustic isolation. First-order dead-time corrections (Johnson *et al.* 1966 b) were made where necessary, giving a maximum correction of 12% to the fourth factorial moment when a 3 μs sample time was used.

Photon-counting distributions were measured through the threshold region for sample times between 3 and 300 μs. In addition the effective coherence time for each intensity was measured using a single-clipping digital autocorrelator at low mean rates (Pike 1969). By interpolation between experimental points obtained using a constant sample time, the factorial moments were compared with the theory of Lax and Zwanziger, for a constant ratio s of the sample time to the effective coherence time of the intensity fluctuations, at given values of the pump parameters p. The results for these second, third and fourth moments of the intensity fluctuations are shown in figure 1. The agreement is seen to be good to a few per cent over almost the entire range. In addition experimental values of the moments for the case $p = -\infty$ are included; these were obtained by scattering laser light from the Brownian motion of a macromolecule (haemocyanin) in solution. The accuracy in this case was better than one per cent over all values. This was limited only by the experimental time taken of approximately 30 s per point.

In figure 2 we show some of the actual photon counting distributions from which the data for figure 1 were obtained. These were taken with a 3 μs sample time and show the change in the distributions as the laser is taken from well below to well

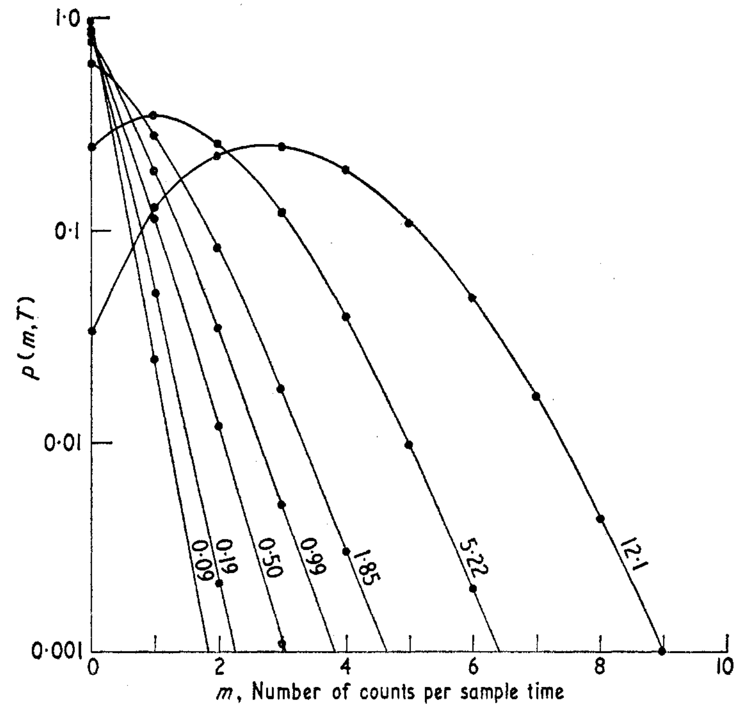

Figure 2. Photon-counting distributions of a laser for different pumping levels from well below to well above threshold recorded with a short (3 μs) sample time. For each distribution the ratio of the intensity to the threshold intensity is given in the figure.

above threshold. Quantitative comparison of such data with theory is always performed through computation of the factorial moments since these functions are independent of beam attenuation (Pike 1969).

The dependence of the second moment of the intensity fluctuation $\langle I^2 \rangle$ upon integration time can be very accurately predicted by use of the following approximate formula:

$$\frac{\langle I^2 \rangle}{\langle I \rangle^2} = 1 + \left\{ \pi \left(\frac{I_0}{I} \right)^2 \left(\frac{p}{\sqrt{\pi}} \frac{I}{I_0} + 1 \right) - 2 \right\} \left(\frac{1}{s} - \frac{1}{s^2} + \frac{e^{-s}}{s^2} \right).$$

This is simply constructed by assuming the intensity fluctuation spectrum to be a single Lorentzian line and using the formula (Jakeman and Pike 1969)

$$\frac{d^2}{dT^2} \left(T^2 \frac{\langle I^2 \rangle}{\langle I \rangle^2} \right) = 2(1 + C\, e^{-s})$$

together with the known form of $\langle I^2 \rangle / \langle I \rangle^2$ at $T = 0$ which defines C.

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Statistical accuracy in the digital autocorrelation of photon counting fluctuations

Abstract. Optical measurements by intensity fluctuation spectroscopy are subject to errors arising from the statistical nature of light and of the photodetection process. We report here the results of a calculation of the expected errors, due to these causes, in linewidth measurements by digital autocorrelation of photon counting fluctuations.

As the methods of intensity fluctuation spectroscopy are applied to a greater variety of problems (see for example Benedek 1968, Pike 1969, Cummins and Swinney 1970) it becomes increasingly important to assess their accuracy as a function of the